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Essays on aging, unemployment, and retirement in a life-cycle model of search and matching in the labor market

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**Essays on aging, unemployment, and retirement in a life-cycle model of
search and matching in the labor market**

by

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A thesis submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

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has met the thesis requirements of Iowa State University

Signatures have been redacted for privacy




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Chapter 1 Introduction

1.1 Background and Motivation

Many developed countries around the world have been witnessing (and will continue to do so for quite a while) a substantial “graying” of their populations. In the United States, in 1850, less than 3% of the population was older than 64. This figure is projected to rise from near 13% in 1990 to 20% in 2050. The ratio of people aged 65 or older to those between 20 and 64 has increased from 0.14 in 1950 to 0.21 in 1997 and will soar to 0.36 by 2030. In the European Union, the ratio of people over age 65 to potential workers (age 15 to 64) to is projected to increase from 0.25 in 2000 to 0.5 in 2050. All this suggests that the age composition of the labor force is rapidly changing. In the United States, the median age of the workforce in the US is expected to be over 40 by 2005 compared to 34.7 in 1979. ¹

Alongside the changing age composition of the labor force and higher life expectancy, many countries have experienced significant reductions in labor force participation rates of older workers. In the United States in 1900 in 65% of men sixty-five years and older were active in the labor force This figure declined to 47% in 1950, 16% in 1993, and less than 3% in 1999. In the UK, the proportion of men aged 60-64 in employment halved from 1968, when 80% were employed, to less than 40% at the end of 1990s. In France, the participation rates among males 55 and older have fallen from 31.5 % to 15% since 1960. More workers are seeking early retirement in the OECD countries. In the United States, the participation rates of workers aged 55 to 64 has declined from 95% in 1880 to 82% in 1940 and 67% in 1990. In

¹ The aging of the population is usually attributed to a decline in fertility rates and an increase in the life expectancy. For example, in 1900, people who survived to age 65 could expect to live another 12 years; by 1995 they could expect to live 17.4 years.

many Western European countries, only 50% of the workers aged 55-59 years are still active in the labor market, and merely 1 in 5 in their early 60s are active. Virtually no one over 65 is in work.

These demographic changes [and government policies that were introduced in response] are creating an unprecedented burden on the current younger generations of working individuals. In the United States for example, the dependency ratio (ratio of the number of SS recipients to the number of workers) is steadily rising from 0.29 in 1997 to about 0.56 by the end of the 75- year forecast. The combined assets of the Old Age, Survivors, and Disability Insurance (OASDI) trust funds are expected to go below the safety level, a year's worth of benefits, in 2032. In the EU countries, the public pension expenditure as a percent of GDP was 10.4% in 2000. This is expected to rise to 13.6% by 2040. In Spain, the public pension expenditure as a percent of GDP is projected to grow from 9.4% in 2000 to 17.3 in 2050. Output in the UK is estimated to decrease by 2% and public finances are likely to be reduced by between £3 billion and £5 billion a year due to increased joblessness.

In response to these recent challenges, many countries have introduced a large variety of pension and social security (hereafter, SS) programs. I summarize some important features of SS programs that are of relevance in understanding present day labor market conditions in OECD countries. First, SS programs have a significant impact on labor market behavior.² Gruber and Wise (1999) attribute the aforementioned large decline in labor force participation rates by older workers to the increasing generosity of public pension programs around the world, as well as to the provision of both early retirement schemes and long-term unemployment insurance programs for older workers.³ Mulligan and Sala-i-Martin (1999)

² There is substantial evidence in the labor literature showing that social security regulations induce retirement (see Coile and Gruber (2000a), Pechman, Aaron and Taussig (1968), Boskin (1986), Boskin and Shoven (1987) and Kotlikoff and Wise (1987). See also the extensive survey on empirical results in Atkinson (1987).

³ Costa (1998) documents that rapidly declining labor force participation rates among men over the age of 65

document that of the 94 countries for which this information was available, 91 induce workers to retire – 74 of them mandate retirement in order to collect benefits while the other 17 encourage retirement through tax and benefit formulas. Coile and Gruber (2000b) find that in the US, “the median male worker faces a small tax on work at ages 55-61, a near zero tax at ages 62-64, and a large tax at ages 65-69.”

The majority of SS programs are pay-as-you-go (PAYG) systems which are financed with payroll taxes, split between both the employer and employee. SS benefits are generally related in some way to the number of years worked (and amount of taxes paid). Benefits tend to increase with lifetime earnings. In some countries (such as Canada, Denmark, or Sweden), the pension has multiple components: a flat payment which is completely unrelated to prior earnings (thereby providing a minimum amount of income for the elderly) and a second component which is tied to previous earnings.

The fact that so many public pension programs encourage older workers to withdraw from the labor market suggests that they are an important vehicle for regulating labor market activity. In fact, a popular view about social security, dating back to its early days of inception, is that it is a means to transfer jobs from older, employed workers to young, unemployed individuals.⁴ President Franklin D. Roosevelt, in one of his “fireside chats” suggested that this would be an important goal for social security: “The program for social security now pending before the Congress is a necessary part of the future unemployment policy of the Government...It proposes, by means of old-age pensions, to help those who have reached the age of retirement to give up their jobs and thus give to the younger generation greater

was witnessed well before any form of social insurance program was established.

⁴ In Bhattacharya, Mulligan, and Reed (2001), I find that although it is possible that policy-induced retirement can promote labor market efficiency, most public pension programs pay the elderly substantially more than labor market search theory implies that their jobs are worth.

opportunities for work and to give all a feeling of security as they look toward their old age...” (Roosevelt, pp.134-5).

Since then, governments have recognized that it is a fallacy to think that economies simply have a fixed number of jobs that can be divided among their citizens: the so-called “lump-of-labor” fallacy. It is no longer believed that shedding older workers will make more room in the economy for younger workers, or that re-employing older workers must be at the expense of opportunities for younger ones. The lump of labor fallacy ignores the flexibility of wages in the labor market. It assumes that the supply of labor is homogenous i.e., a young worker is a perfect substitute for an old worker and therefore has no long-term effect on labor demand and firm entry.

Finally, as pointed out by Gruber and Wise (1999), many countries have introduced a variety of early retirement schemes and long-term unemployment insurance programs that allow for older, displaced workers to retire before the normal retirement age. These programs are extremely generous, providing as much as 60-70% of previous income and have also been introduced to combat unemployment problems facing younger workers. For example, in France, the “contrat de solidarite” recognized the “double need to encourage 55-59 year-old workers to stop work and bring young workers into the labor market, as rising youth unemployment was a growing concern to society as a whole.” Furthermore, a precondition to receiving unemployment benefits for people over the age of 55 is that they stop “seeking employment”.⁵

Recent evidence suggests that an individual’s age has important implications for their labor market experiences. First, the unemployment rate of younger workers is higher than that of older workers. For example, in 1987, the unemployment rate for male workers under

⁵ For more details, see the OECD (1995) study on “The Labor Market and Older Workers”, Gruber and Wise (1999), and the discussion in Bhattacharya, Mulligan, and Reed (2001).

the age of 25 in the US was 12.6% compared to 4.8% for those over the age of 25. In France during the same period, the unemployment rate for younger workers was 19.6%, but only 6.4% for older workers. Second, the incidence of long-term unemployment is much higher for older workers. In the United States in 1996, while the incidence of long-term unemployment for the overall labor force (15 to 64 years) was only 9.3%, this number rises to 14.6% for older workers (between the ages of 45 and 64). In France, the incidence of long-term unemployment among the entire workforce is much higher (39.5%); that for workers between the ages of 45 and 64 is as high as 62.0%.

Furthermore, older workers who experience involuntary job loss are more likely to become permanently separated from their employers. O'Leary and Wandner (2000) find that while less than 10% of displaced workers under the age of 55 permanently exit the labor force, more than 25% of workers between the ages of 55 and 64 and almost half of workers over the age of 65 opt for retirement instead of searching for alternative sources of employment upon displacement. Additionally, as discussed in Jacobson et. al. (1993), upon finding employment displaced workers receive wages that are about 25% lower than in their previous job.

There are many explanations for why older workers find it harder to find jobs. An important explanation due to Oi (1962) and further advanced by Hutchens (1986), is that the initiation of an employment relationship requires firms to incur fixed costs thereby increasing the incentive to hire younger workers. It may also be true that firms choose not to hire older workers because they are relatively less productive.⁶

A quick summary of the above discussion is in order. First, in almost all countries, the labor force is getting older. Secondly, labor force participation rates of older workers

⁶ In a two-period overlapping generations model, Pissarides (1992) considers the impact of lost productivity for older workers resulting from long-term unemployment. Laing, Palivos, and Wang (2001), in a model with vintage human capital, assume that workers born in later periods will be less productive than newly-born workers due to economic growth.

have been steadily declining during the post-war period. Thirdly, many public policies affect the labor market participation decision of older workers in that they encourage retirement. Fourthly, an individual's age plays a significant role in determining their labor market status. Young workers are more likely to be unemployed (but not for long); older workers who lose their jobs are more likely to remain unemployed for a long period of time. Many older workers may become discouraged about the prospects of job search and choose to retire. Those who find new jobs will earn less than in their previous positions.

1.2 Relevant Literature

Two dominant (and fairly divergent) approaches to modelling the labor market are to be found in the literature. The Walrasian paradigm endows workers and firms with perfect information about the labor market conditions; in particular every worker is assumed to know the wage offered by each firm in the economy. At the other extreme is the search paradigm, which generally assumes that job-seeking workers interact with the firms in a labor market characterized by equilibrium unemployment. The two basic features of the Walrasian approach are full employment and equality between wages and the marginal product of labor. Search models, on the other hand, allow for frictional unemployment and typically yield wages below the marginal product of labor. Search frictions break the link between marginal product and wages, and introduce rent-sharing between workers and firms.

The above review of the current demographic trends strongly suggests that to understand the implications of these trends on the aggregate labor market, a model with equilibrium unemployment is imperative. Diamond (1982), Mortensen (1982), and Pissarides (1990) analyzed unemployment in a matching framework. These models postulate the existence of a matching function that maps measures of unemployment and vacancies into a

measure of matches. A match pairs a worker and a firm who then have to bargain on how to share the match surplus. The outcome of the matching framework is in general not efficient. There are either too many or too few vacancies posted in equilibrium. Therefore these models, apart from incorporating frictional unemployment, address the general equilibrium consequences of congestion in the labor market and its effect on unemployment and wages through the process of bargaining. The search model has been used extensively in the literature to understand the implications of job heterogeneity. Acemoglu (1997), Bertola and Caballero (1994), and Davis (1995) explore the matching models where the heterogeneity on the job supply side is negotiated through a single matching function. Albrecht and Vroman (2002) investigate a labor market in which workers differ in their abilities and jobs differ in their skill requirements. This addresses the issues of differences in skills across workers and differences in skill requirements. The existing literature with equilibrium unemployment mainly focuses on the heterogeneities in the labor market via skill differences.

The effect of age and a worker's position on the life-cycle has strong effects on an individual's labor market participation as discussed in the review above. Bhattacharya and Reed (2001)⁷ is an early attempt at formalizing some of these issues. The life-cycle of workers explicit in their overlapping-generations setup in conjunction with the search frictions in their labor market formulation produces a rich environment in which both young and old workers may find themselves contemporaneously competing for the same jobs. It is this meeting of the young and the old in the labor market that produces interesting consequences for both the age groups at the individual level. B&R also attempt to capture the general equilibrium effects of these individual-level consequences by exploring policies that affect the labor-force participation decision of workers, and through this channel, firm entry.

⁷ Hence forth referred to as B&R.

This thesis essentially starts off where B&R left. In particular, it is designed to address one major lacuna in the B&R paper. In their setup, policies that induce the old with jobs to retire would be unambiguously disastrous for the economy, and here's why. Their life-cycle model differentiates between a young and an old worker by recognizing each worker's position on the life-cycle. A young worker [who has two periods left to live] can be matched with a firm for two periods while an old worker [who has only one period left to live] can at most be matched with a firm for one period. Inducing the old with jobs to retire would therefore equate job durability over different points in the life-cycle thereby undermining the entire focus of the model. In short, the B&R model is ill-suited to study retirement by old people who have jobs.

A study of the labor market most often addresses the issue of retirement. Retirement is at times referred to as the "golden age" and the "leisure years". The French call it "The Third Age" or the age of living (which follows the ages of learning and working). Given the life-cycle frame-work, where the young and the old interact in the labor market to search for jobs, a distinction between the old workers is essential to model retirement. This distinction between the old workers enables policy to target that group of old workers who have some option outside the labor market. Leisure seems to be a natural choice as people who prefer leisure are more likely to withdraw from the labor market. Recent trends in the labor market indicate that an increasing number of retirees are citing a preference for leisure as their main motivation for leaving the labor force. Costa documents that among the men who retired at age 65, in 1941-1951 only 3% gave leisure to be their reason for retiring. This number rose to 17% in 1963 and 48% in 1982.

Retirement is a period of life that is anticipated and enjoyed by most who leave the work force. When retiring people always feel, "We have served our time; now we deserve to

do some things that we have put off". It comes as no surprise that a gerontologist reported that "In their fifties many men feel an urgency to do things that their occupational lives had made it difficult". This suggests that preference for leisure almost comes as a shock towards the end of an individual's work life and motivates workers to leave the work force. Therefore, in this study, a worker's preference for leisure (high/low) is revealed at the end of the first period and before the beginning of the second period.

Moreover, Costa documents that expenditure elasticities of recreational goods has fallen from more than two at the beginning of the century to slightly more than one today (earlier a luxury but now more or less normal good). Expenditure elasticities have fallen due to two reasons: technological change that has lowered the price of recreation and increase in the public provision of goods that are complementary to recreational goods. This implies that recreation is now attractive and inexpensive and that decreases in income through policy intervention may not lead to substantial increases in labor force participation.

In addition, there is a heightened concern for displaced workers. The frequency of job loss among workers over the age of 50 has risen disproportionately in recent years. Farber (1996) shows that the three-year probability of job loss among workers 55 and over rose from 11% in 1981 to more than 16% in 1993. Between 1981 and 1993, workers 55 to 64 experienced the largest increase in job loss probabilities for any age group. The economic impact for these workers is potentially quite severe as older workers typically have high pre-displacement job tenure and are much less likely than younger workers to be re-employed. The Congressional Budget Office (1993) finds that workers aged 55 to 69 are more than twice as likely to be out of the labor force following a job loss than younger workers. Among workers over 60 years old, more than half have left the labor force following a job loss. Chan and Stevens (1999) find that a job loss at age 50 or above has substantial and long-lasting

employment effects, and that these effects vary with age at which job loss occurs. Displaced workers in their 50s are estimated to have a three quarter chance of returning to work within two years after a job loss, whereas for a 62-year-old job loser, the probability is less than a third.

B&R address the issue of job separation in their model but fail to distinguish the old separated workers from the old never-before employed workers. A match between a young worker and a firm can dissolve after the end of the first period. The old workers who were matched in the previous period are the separated workers. These workers re-enter the labor market in search for jobs. B&R at this point do not distinguish between an old separated worker and an old never-before employed workers. Therefore, these two workers draw equal wages in the labor market inspite of differing in their employment histories. In this study, these two types of old workers are differentiated through the hiring costs incurred by the firms. The cost of hiring an old separated worker is lower that of the never-before employed worker. This implies that the training acquired by the separated worker in his previous job is in some way transferable. Therefore, these two workers in this model draw different wages in the labor market.

1.3 Thesis Organization

The principal aim of this research is to build a dynamic general equilibrium model to study the implications of inter-generational conflicts on labor market. An overlapping generations model with finitely-lived agents explicitly separates workers based on their position in the life-cycle. Moreover, the old workers are differentiated not only on the basis of their employment histories but also through their preference for leisure. A fraction δ of old agents get utility from leisure while the remainder do not care about leisure at all. It is

assumed that the utility for the low leisure type equals zero. The intensity of worker preference for leisure is revealed to a worker at the end of the first period of life. The preference for leisure, when old, affects a worker's wage bargaining with the firm both when young and when old since it directly affects their threat points.⁸ Thus in the labor market, at any point in time, there are workers with seven different possible histories: the young, the old employed (with unbroken matches from the previous period) with a high/low leisure preference, the old who got separated from their previous match with a high/low leisure preference, and the old never-before employed with a high/low leisure preference.

In addition, the labor market in the real world is characterized by unemployment. This aspect is captured by modelling the labor market in a search and matching framework. There are two agents in the market, workers and firms. Both agents spend resources before job creation and production takes place. The two agents are brought together via a matching technology. The matching technology is a function of the total number of workers seeking employment and total number of firms seeking employees. Therefore, an additional unemployed worker reduces the probability of a worker getting matched with a firm but increases the probability of a firm getting matched with a worker. The trading externality is a key feature of the search and matching models. Given the life-cycle model, a match with a young worker is durable for a maximum of two periods. A match with an old worker can survive only one period. The durability of a match has implications for the profits accruing to a firm. For example if a match is durable for two periods, then the firm has to incur search and hiring costs after two periods. But if the match is durable only for one period, then the firm

⁸ Pissarides (2001) introduces a notion of leisure in his discussion of labor market participation. In his analysis, he assumes that all workers receive some benefits from leisure, but the value of leisure varies across individuals. In contrast to his analysis, I model that the benefits of leisure may accrue over the course of the lifecycle and therefore will affect the retirement decisions of older workers. I also show how these additional aspects of aging affect aggregate labor market behavior through their interactions with wages at each stage of the cycle and vacancy creation.

has to incur these costs in the second period itself. The total surplus from a match is equally shared between the two agents. Nash Bargaining determines the equilibrium wages.

Using this set-up, Chapter 2 characterizes in detail the "Baseline Case". Here all the seven workers participate in the labor market. Given the pattern of labor market participation, the main aim of the Baseline case is to unravel the interactions between the young and old workers when workers are differentiated through their position in the life-cycle and their preference for leisure. In addition, this set-up would enable comparison with the Baseline case of B&R. This, in turn would explicitly indicate the effect on the labor market of introducing heterogeneity among the older workers through differing employment histories and leisure preference. Chapter 2 describes the steady state equilibrium, discusses a numerical benchmark example and compares the results to that of B&R and further investigates the effect of some exogenous variables on the aggregate labor market and welfare.

Chapter 3 characterizes the "True-Retirement" Case. Here the old employed workers with high preference for leisure withdraw from the labor market. Therefore, at any point in time, there are six types of workers actively participating in the labor market. This case highlights the effect on the labor market of the withdrawal of workers that are currently employed but have an option outside the market. It is this group of workers that are the focus of various labor market policies. An analysis of this case therefore, gives us useful insights because the withdrawal is voluntary and not policy induced. Chapter 3 further compares the outcome of the True Retirement case with the Baseline case and discusses the welfare implications.

Chapter 4 discusses the main findings of the model and concludes the study.

Chapter 2

Baseline Case

This chapter describes the model in general and focuses mainly on the Baseline case. In this case all the seven different types of workers actively participate in the labor market. Given the pattern of labor market participation, the Baseline case focuses on the impact of life-cycle of workers and their preference for leisure on the aggregate labor market. Later in this chapter, the results of a benchmark numerical experiment are compared to B&R in order to highlight the differences in the two models and their outcomes on the labor market.

2.1 Environment

The economy consists of an infinite sequence of two-period lived overlapping generations. It is populated by a continuum of two types of agents, workers and firms. These agents are risk neutral. There is no growth in the population sizes of these agents. Workers live only for two periods. Consequently, in any period, there are two types of workers - young and old. Each generation of workers is of equal measure, $\frac{1}{2}$, so that the total population of workers is equal to 1 in each period. At birth, all workers are unemployed. There are no assets or saving instruments. All income earned is consumed in the same period. Towards the end of the first period of life, workers are subject to a two-state idiosyncratic leisure shock which determines for them their relative preference for leisure in the second period of their lives.

Technologically identical firms of a single industry produce a homogenous consumption good each period using labor as the factor of production in a partial-equilibrium framework. Production is the result of a pair-wise matching between one worker and a firm. Trade in the labor market is a decentralized economic activity. It is uncoordinated, time consuming and costly for both agents. The agents do not coordinate their activities but instead behave as

individual competitors. Both agents spend resources before job creation and production takes place. This trade is non-trivial if there are heterogeneities and frictions in the labor market. In my model, there are two main sources of heterogeneity: position in the life-cycle (i.e., age) and old-age preference for leisure. All agents share the same discount factor β . In the model, a pair-wise random matching process between firms and workers (of all ages) and a bargaining process determine the wages of the workers and net revenue of the firms.

Matches between a firm and a worker potentially last two periods if the worker is young when recruited. If an old worker is hired, then the firm will have to incur hiring and other costs the following period. Thus the life-cycle of agents has implications for job durability and on the total surplus created. B&R identified a source of inefficiency here: young people with two periods to live may remain unemployed even when the old with one period to live may get hired. My model extends their analysis by identifying yet another source of inefficiency. Old workers with jobs with a high preference for leisure may co-exist alongside young workers with no jobs and no desire for leisure. Put starkly, there are old workers who have jobs but who would rather not work [and hence have to be compensated for working], and then there are workers who would rather work but are unemployed.

2.2 Timeline

This section describes the basic features of the model and the sequence of events that occur in the labor market. Let t be the time index. The labor market opens at the beginning of each period. The unemployed workers, young and old, choose whether to search for vacancies or not. When a worker, young or old, decides to search, he incurs an upfront cost s . If firms decide to enter the market to search for employees, then they incur an upfront cost a . Prior to the opening of the labor market, old workers have knowledge about their leisure preference.

The firms know the probability distribution with which an old worker can be of high or low leisure type and given this information they decide to enter the labor market based on their expected profits. The firms cannot discriminate between workers based on either their age or their preference for leisure.

Let $U_t (F_{v,t})$ denote the total mass of unemployed workers (unfilled vacancies) at the start of date t . Unemployed workers and unfilled vacancies are brought together via a stochastic matching process. Any unemployed worker faces the probability α_t of getting matched with an unfilled vacancy in period t . A firm faces the probability θ_t of locating a worker. At the end of any period, the match formed at the start of the period may dissolve with an exogenously given probability b . A match, therefore survives for a minimum length of one period and a maximum length of two periods. The probability of meeting a given type of worker (young or old), will depend on the participation of each type ($u_{y,t}$ or $u_{s,t}$ or $u_{o,t}$). The probability of finding a young unemployed worker is $\theta_t \tilde{u}_{yt}$, where \tilde{u}_{yt} that equals $\frac{u_{y,t}}{U_t}$, is the ratio of young unemployed workers to the total mass of unemployed comprising of the young, old separated and never-before employed old. Similarly, the probability of meeting an old separated worker is $\theta_t \tilde{u}_{st}$, where $\tilde{u}_{st} \equiv \frac{u_{s,t}}{U_t}$ and the probability of meeting an old never-before employed worker is $\theta_t \tilde{u}_{ot}$, where $\tilde{u}_{ot} \equiv \frac{u_{o,t}}{U_t}$.

In the equilibrium of the baseline case, at the start of any date t , both young and old workers participate in the labor market. All the young are unemployed. The old workers fall under three different employment histories: employed, separated and never-before employed. The employed old are the workers who did not get separated from their previous job, the separated are the workers who got separated with an exogenously given probability and the never-before employed are the workers who did not get matched when young. Therefore, at the start of date t , an old worker is employed with probability $\alpha_{t-1} (1 - b)$, separated but

employed when young with probability $\alpha_{t-1}b$ and never-before employed with probability $1 - \alpha_{t-1}$. Apart from different employment histories, the old workers are further differentiated by their preference for leisure. An old worker can have a high (low) preference for leisure with an exogenously given probability δ ($1 - \delta$). Consequently, at any date t there are seven types of workers in the economy.

Once a match is formed and job creation takes place, production occurs. Let p denote the exogenously determined market value of the firm's output. Matches with new hires (young or never-before employed old) require the firm to incur the cost of training h so the net output is $(p - h)$, while matches with an old separated worker require it to incur a lower cost of training h_s so the net output is $(p - h_s)$ and the net output of a firm with a retained worker is p . Comparing the net surplus between a match with a new hire vis-a-vis a separated old worker indicates that the surplus to a firm is more when it gets matched with a separated worker. Therefore, employment histories have an impact on the profitability of firms and the labor market rewards workers with previous experience. This assumes that skills are transferable across firms.

Any match formation potentially creates a surplus for both the agents, workers and firms. It is assumed that the total surplus is equally shared by both the agents.

2.3 Worker's Payoff

Workers constitute the labor supply and are the only input used in the production process. Their decisions to enter the labor market is based on their ex-ante expected utilities. Let l_h (l_l) be the utility that a worker with high (low) preference derives from leisure. Let J_{yt} be the expected lifetime utility of a young worker if he chooses to seek employment and w_{yt} be the wage paid to a young worker in period t . Let $J_{o,t}^{eh}$ ($J_{o,t}^{el}$) be the expected lifetime

utility of an old worker with high (low) preference for leisure who begins period t employed at a firm and $w_{o,t}^{eh}$ ($w_{o,t}^{el}$) be the wage paid. Let $J_{o,t}^{sh}$ ($J_{o,t}^{sl}$) denote the expected lifetime utility of an old worker with high (low) preference for leisure who was employed when young but got separated and begins period t unemployed and $w_{o,t}^{sh}$ ($w_{o,t}^{sl}$) be the wage paid. Finally, let $J_{o,t}^{uh}$ ($J_{o,t}^{ul}$) denote the expected lifetime utility of an old worker with high (low) preference for leisure who was unemployed when young and $w_{o,t}^{uh}$ ($w_{o,t}^{ul}$) be the wage paid. The following value functions govern the expected present discounted surplus of workers in each state:

$$J_{yt} = -s + \alpha_t [w_{yt} + \beta(1-b) \{ \delta (\max(J_{o,t+1}^{eh}, l_h)) + (1-\delta) (\max(J_{o,t+1}^{el}, l_l)) \}] \quad (2.1)$$

$$+ \beta b \{ \delta (\max(J_{o,t+1}^{sh}, l_h)) + (1-\delta) (\max(J_{o,t+1}^{sl}, l_l)) \}$$

$$+ (1-\alpha_t) \beta \{ \delta (\max(J_{o,t+1}^{uh}, l_h)) + (1-\delta) (\max(J_{o,t+1}^{ul}, l_l)) \}$$

$$J_{o,t}^{eh} = w_{o,t}^{eh} \quad (2.2)$$

$$J_{o,t}^{el} = w_{o,t}^{el}$$

$$J_{o,t}^{sh} = -s + \alpha_t w_{o,t}^{sh} + (1-\alpha_t) l_h$$

$$J_{o,t}^{sl} = -s + \alpha_t w_{o,t}^{sl} + (1-\alpha_t) l_l$$

$$J_{o,t}^{uh} = -s + \alpha_t w_{o,t}^{uh} + (1-\alpha_t) l_h$$

$$J_{o,t}^{ul} = -s + \alpha_t w_{o,t}^{ul} + (1-\alpha_t) l_l$$

When a young worker seeks employment he must incur a search cost of s . With probability α_t , he gets matched and earns a wage w_{yt} . In the following period ($t+1$), he may be still matched with the same firm with probability b . Depending on his preference for leisure that is revealed before the beginning of the second period, he faces the choice of retaining the job or exiting the labor market to enjoy leisure. If instead the match dissolves after the

first period with probability $(1 - b)$, the worker re-enters the labor market in period $(t + 1)$ as a separated worker after evaluating his expected utility from search versus leisure. With probability $(1 - \alpha_t)$, the young worker does not get matched when young and is unemployed at the start of $t + 1$. He chooses to seek employment if the expected utility from seeking work exceeds the utility from leisure. The workers decision to enter the labor market to seek employment in the second period is made prior to the opening of the market. In addition, an old worker is of high (low) leisure type with probability δ $(1 - \delta)$ and this too is revealed to the worker prior to the opening of the labor market.

In the Baseline case, under the assumption that all the workers participate in the labor market, the following inequalities are imposed on the model: $J_{yt} > \beta (\delta l_h + [1 - \delta] l_l)$, $J_{o,t}^{eh} > l_h$, $J_{o,t}^{el} > l_l$, $J_{o,t}^{sh} > l_h$, $J_{o,t}^{sl} > l_l$, $J_{o,t}^{uh} > l_h$, and $J_{o,t}^{ul} > l_l$. It is also assumed the workers with low preference for leisure do not derive any utility from leisure, i.e., $l_l = 0$. Rewriting the above equation (2.1) after substituting equation (2.2) in the *steady state* gives

$$\begin{aligned} J_y = & -s + \alpha [w_y + \beta (1 - b) \{ \delta w_o^{eh} + (1 - \delta) w_o^{el} \} \\ & + \beta b \{ \delta (-s + \alpha w_o^{sh} + (1 - \alpha) l_h) + (1 - \delta) (-s + \alpha w_o^{sl}) \}] \\ & + (1 - \alpha) \beta \{ \delta (-s + \alpha w_o^{uh} + (1 - \alpha) l_h) + (1 - \delta) (-s + \alpha w_o^{ul}) \} \end{aligned}$$

The total mass of unemployed workers in the Baseline case is $U_t = u_{y,t} + u_{s,t} + u_{o,t}$, where $u_{y,t}$ is the young unemployed that is $\frac{1}{2}$ by assumption, $u_{s,t}$ is the separated unemployed and $u_{o,t}$ and is the unemployed old. In the *steady state*, $u_s = \frac{\alpha b}{2}$, $u_o = \frac{1 - \alpha}{2}$ and $U = \frac{1}{2} + \frac{\alpha b}{2} + \frac{1 - \alpha}{2} = \frac{2 - \alpha(1 - b)}{2}$ and hence the fraction of each type of unemployed in the total unemployed population is $\tilde{u}_y \equiv \frac{1}{2 - \alpha(1 - b)}$, $\tilde{u}_s = \frac{\alpha b}{2 - \alpha(1 - b)}$ and $\tilde{u}_o = \frac{1 - \alpha}{2 - \alpha(1 - b)}$.

2.4 Firm's Payoff

The production technology exhibits constant returns to scale with labor as the only input. Each firm employs at most one worker. Therefore, a firm can be matched with either a young or an old worker. Production precedes matching and each employed worker produces p units of output. Firms begin each period in one of the two possible states. They have a vacancy or they are matched from the previous period with an old worker and will have a vacancy for sure the following period. Let $\pi_{v,t}$ be the expected lifetime profits of a firm that has an unfilled vacancy and $\pi_{f,t}^h$ ($\pi_{f,t}^l$) be the expected lifetime profits of a firm that is matched with an old worker with high (low) preference for leisure. The firm knows the probability with which an old worker can be of high or low leisure type. The following Bellman equations describe the expected discounted profits of a firm in each state:

$$\begin{aligned}
 \pi_{v,t} = & -a + \theta_t \tilde{u}_{yt} [(p - h - w_{yt}) + \beta b \pi_{v,t+1}] \\
 & + \beta (1 - b) \left\{ \delta \begin{pmatrix} \pi_{f,t+1}^h & \text{if } J_{o,t+1}^{eh} > l_h \\ \pi_{v,t+1} & \text{otherwise} \end{pmatrix} + (1 - \delta) \begin{pmatrix} \pi_{f,t+1}^l & \text{if } J_{o,t+1}^{el} > l_l \\ \pi_{v,t+1} & \text{otherwise} \end{pmatrix} \right\} \\
 & + \theta_t \tilde{u}_{st} \left[\delta \begin{pmatrix} (p - h_s - w_{o,t}^{sh}) & \text{if } J_{o,t}^{sh} > l_h \\ 0 & \text{otherwise} \end{pmatrix} + (1 - \delta) \begin{pmatrix} (p - h_s - w_{o,t}^{sl}) & \text{if } J_{o,t}^{sl} > l_l \\ 0 & \text{otherwise} \end{pmatrix} \right. \\
 & \quad \left. + \beta \pi_{v,t+1} \right] \\
 & + \theta_t \tilde{u}_{ot} \left[\delta \begin{pmatrix} (p - h - w_{o,t}^{uh}) & \text{if } J_{o,t}^{uh} > l_h \\ 0 & \text{otherwise} \end{pmatrix} + (1 - \delta) \begin{pmatrix} (p - h - w_{o,t}^{ul}) & \text{if } J_{o,t}^{ul} > l_l \\ 0 & \text{otherwise} \end{pmatrix} \right. \\
 & \quad \left. + \beta \pi_{v,t+1} \right] + (1 - \theta_t) \beta \pi_{v,t+1}
 \end{aligned} \tag{2.3}$$

$$\pi_{f,t}^h = (p - w_{o,t}^{eh}) + \beta\pi_{v,t+1} \quad (2.4)$$

$$\pi_{f,t}^l = (p - w_{o,t}^{el}) + \beta\pi_{v,t+1}$$

When a firm posts a vacancy, it incurs a cost a . With probability $\theta_t \tilde{u}_{yt}$, it gets matched with a young worker and earns a net revenue of $(p - h - w_{yt})$. The firm will have a vacancy the following period with probability b , if the match dissolves. With probability $(1 - b)$, the match does not dissolve. Depending on the worker's preference for leisure, the worker may either work or withdraw therefore implying that the firm will have a filled vacancy or a vacancy respectively in the following period. If the firm has a filled vacancy, the expected discounted profits are $\pi_{f,t+1}^h (\pi_{f,t+1}^l)$ with probability $\delta ((1 - \delta))$. With probability $\theta_t \tilde{u}_{st}$, the firm gets matched with a separated worker and earns a net revenue of $(p - h_s - w_{o,t}^{sh})$ with probability δ and $(p - h_s - w_{o,t}^{sl})$ with probability $(1 - \delta)$. The firm then has an unfilled vacancy for sure in the next period. The firm gets matched with a worker who was unemployed in the previous period with probability $\theta_t \tilde{u}_{ot}$ and earns a net revenue of $(p - h - w_{o,t}^{uh})$ with probability δ and $(p - h - w_{o,t}^{ul})$ with probability $(1 - \delta)$. The firm has an unfilled vacancy in the following period. In period t if the firm does not get matched with probability $(1 - \theta_t)$, then it will have an unfilled vacancy in the following period. Firms that begin a period with a filled vacancy do not incur any hiring costs and their net revenues are $p - w_{o,t}^{eh}$ or $p - w_{o,t}^{el}$ depending on workers' leisure preference. These firms will have an unfilled vacancy in the following period. Note that the expected profits of a firm depend on the employment histories of the workers and their preference for leisure.

Focussing on the Baseline case in the *steady state* where all the workers choose to seek employment, equation (2.3) can be re-written after substituting equation (2.4) as

$$\begin{aligned} \pi_v = & \frac{1}{1 - \theta \tilde{u}_y \beta (b + \beta [1 - b]) - \theta \tilde{u}_s \beta - \theta \tilde{u}_o \beta - (1 - \theta) \beta} \{ -a + \theta \tilde{u}_y [(p - h - w_y) \\ & + \beta (1 - b) \{ \delta (p - w_o^{eh}) + (1 - \delta) (p - w_o^{el}) \}] \\ & + \theta \tilde{u}_s [\delta (p - h_s - w_o^{sh}) + (1 - \delta) (p - h_s - w_o^{sl})] \\ & + \theta \tilde{u}_o [\delta (p - h - w_o^{uh}) + (1 - \delta) (p - h - w_o^{ul})] \} \end{aligned}$$

Let F_t be the total number of existing firms at the start of date t . Let $F_{v,t}$ denote the total number of firms with an unfilled vacancy at the start of date t , and $F_{f,t}$ be the total number of firms with a filled vacancy at the start of date t . If the firm has a vacancy at t , it can find itself in one of three possible situations at $t + 1$. First, *a*) it does not find a worker at t and hence it will have a vacancy at $t + 1$, or *b*) it finds a young worker this period; this worker gets separated with probability b_t in which case the firm will have a vacancy the next period, or the worker does not get separated with probability $(1 - b_t)$ in which case it will not have a vacancy at $t + 1$, or *c*) it finds an old worker this period in which case the firm will definitely have a vacancy at $t + 1$. Then it follows that $F_{v,t+1} = (1 - \theta) F_{v,t} + \theta \tilde{u}_{y,t} b F_{v,t} + F_{f,t} + \theta (\tilde{u}_{s,t} + \tilde{u}_{o,t}) F_{v,t}$ while $F_{f,t+1} = \theta \tilde{u}_{y,t} (1 - b) F_{v,t}$ and accounting restrictions require that $F_{f,t} + F_{v,t} = F_t$. In the *steady state* in the Baseline Case, the number of firms in the two states are $F_v = \frac{\alpha}{2\theta} [2 - \alpha (1 - b)]$; $F_f = \frac{\alpha(1-b)}{2}$ and thus the total number of firms is given by $F = \frac{\alpha}{2} \left[\frac{2 - \alpha(1-b)}{\theta} + (1 - b) \right]$.

2.5 Matching

Workers enter the labor market to seek employment and the firms with unfilled vacancies enter to seek employees. Prior to production, the firms and workers are brought

together via the matching technology. The matching technology describes the total number of matches, $m_t = \mu M(U_t, F_{v,t})$, that are formed at the beginning of each period. It is a function of the aggregate number of workers looking for a job and the number of firms looking for workers. Since α_t represents the probability that an unemployed worker will find a vacancy in period t and θ_t is the probability that an unfilled vacancy will find an unemployed worker, it follows that the total number of workers who find employment ($\alpha_t \cdot U_t$) must equal the total number of firms that filled their vacancies ($\theta_t \cdot F_{v,t}$):

$$\alpha_t \cdot U_t = \theta_t \cdot F_{v,t}$$

It is important to note that α_t and θ_t are determined in equilibrium, and that both workers and firms take them as given when making their decisions to participate in the labor market. The ratio between workers and firms, $q = \frac{F_v}{U}$, is commonly called the tightness of the labor market. Note that $m_t = \theta_t \cdot F_{v,t}$ and this gives the following equalities:

$$\alpha_t \cdot U_t = \theta_t \cdot F_{v,t} = m_t = \mu M(U_t, F_{v,t})$$

The matching technology is assumed to be the Cobb-Douglas. In the steady state $m(U, F_v) = \mu(U)^\phi (F_v)^{1-\phi}$ where $\phi \in [0, 1)$. This implies that the probability that a firm gets matched with a worker is $\theta(q) = \mu(q)^{-\phi}$ and the probability that a worker gets matched with a firm is $\alpha(q) = \theta(q)q$. The tighter the market is, the lower is the probability of a firm getting matched with a worker. The matching function is increasing in both its arguments, concave and homogenous of degree 1. An increase in either the number of unemployed workers or unfilled vacancies increases the number of matches each period, but at a decreasing rate. The extent of lubrication in the labor market is given by $\mu > 0$; ceteris paribus, more matches occur when μ is higher. Note that the matching function also exhibits constant elasticities

$$\frac{\partial m(U, F_v)}{\partial U} \frac{U}{m(U, F_v)} = -\theta'(q) \frac{q}{\theta(q)} = \phi$$

Thus ϕ is the elasticity of the matching function with respect to the measure of unemployment and the negative of the elasticity of the probability that a firm finds a worker with respect to the tightness of labor market. Therefore a high ϕ implies that an additional vacancy has a larger negative impact on all firms' probability of getting matched and therefore filling their vacancy. A high ϕ would also imply that an additional worker will have a larger positive impact on all firms probability of filling their vacancy.

2.6 Bargaining and Wage Determination

The friction built into the job-firm matching process creates the possibility that a firm may remain unproductive or a worker may remain unemployed in any period. Firms and workers must therefore weigh the implications of finding themselves in these states and their outside options when bargaining over their share of current and future surplus produced. In this set-up, the total surplus created (when a firm is matched with a worker) will depend on the worker's age, employment history, and preference for leisure.

Young worker

A young worker seeks employment and with probability α he gets matched with a firm. Let w_{yt} be the wage he gets in the first period. In the following period, with probability $(1 - b)$ the match does not dissolve and the worker remains employed with the same firm. If the worker decides to work instead of enjoying leisure, his expected utility is $\beta\delta J_o^{eh}$ or $\beta(1 - \delta) J_o^{el}$ depending on his leisure preference. If the match dissolves with probability b at the end of the first period, the worker re-enters the labor market if his expected utility ($\beta\delta J_o^{sh}$ or $\beta(1 - \delta) J_o^{sl}$) from seeking employment exceeds the utility from leisure. If instead the young worker does not find employment and re-enters the labor market in the following period, his expected discounted utility would be $\beta\delta J_o^{uh}$ or $\beta(1 - \delta) J_o^{ul}$ depending on his

leisure preference. Thus, a young worker's surplus from finding employment in period t is given by the following equation.

$$\begin{aligned} w_{yt} + \beta(1-b) \{ & \delta (\max(J_{o,t+1}^{eh}, l_h)) + (1-\delta) (J_{o,t+1}^{el}) \} \\ & + \beta b \{ \delta (\max(J_{o,t+1}^{sh}, l_h)) + (1-\delta) (J_{o,t+1}^{sl}) \} \\ & - \beta \{ \delta (\max(J_{o,t+1}^{uh}, l_h)) + (1-\delta) (J_{o,t+1}^{ul}) \} \end{aligned}$$

For a firm, the surplus in the first period from hiring a young worker is the output net of the hiring and wage costs. If the worker is separated then the firm has a vacancy the next period and its expected discounted profits in period t are $\beta\pi_{v,t+1}$. If the worker is not separated and decides to work, then it earns expected discounted profits of $\beta\delta\pi_{f,t+1}^h$ or $\beta(1-\delta)\pi_{f,t+1}^l$ in period t . If the worker is not separated but decides not to work, then it has a vacancy the next period. If the firm would not have found a worker in period t , it would earn expected profits of $\beta\pi_{v,t+1}$ as of period t . Thus the gains from trade accruing to a firm from hiring a young worker in period t is given by

$$\begin{aligned} & (p - h - w_{yt}) + \beta b \pi_{v,t+1} \\ & + \beta(1-b) \left\{ \delta \begin{pmatrix} \pi_{f,t+1}^h & \text{if } J_{o,t+1}^{eh} > l_h \\ \pi_{v,t+1} & \text{otherwise} \end{pmatrix} + (1-\delta) \pi_{f,t+1}^l \right\} - \beta\pi_{v,t+1} \end{aligned}$$

Given that the match between a young worker and a firm results in surplus, this total surplus is shared equally between the two agents. This implies that:

$$\begin{aligned}
& w_{yt} + \beta(1-b) \{ \delta (\max(J_{o,t+1}^{eh}, l_h)) + (1-\delta) (J_{o,t+1}^{el}) \} + \beta b \{ \delta (\max(J_{o,t+1}^{sh}, l_h)) \\
& \quad + (1-\delta) (J_{o,t+1}^{sl}) \} - \beta \{ \delta (\max(J_{o,t+1}^{uh}, l_h)) + (1-\delta) (J_{o,t+1}^{ul}) \} \\
& = (p - h - w_{yt}) + \beta b \pi_{v,t+1} + \beta(1-b) \left\{ \delta \begin{pmatrix} \pi_{f,t+1}^h & \text{if } J_{o,t+1}^{eh} > l_h \\ \pi_{v,t+1} & \text{otherwise} \end{pmatrix} + (1-\delta) \pi_{f,t+1}^l \right\} \\
& \quad - \beta \pi_{v,t+1}
\end{aligned} \tag{2.5}$$

In the Baseline Case, the wage for a young worker in the *steady state* is therefore:

$$\begin{aligned}
w_y & = \frac{(p-h)}{2} + \frac{1}{2} (\beta(1-b) (\delta \pi_f^h + (1-\delta) \pi_f^l)) - \beta \pi_v (1-b) (1-\beta) \\
& \quad - \frac{1}{2} (\beta(1-b) (\delta J_o^{eh} + (1-\delta) J_o^{el})) - \frac{1}{2} (\beta b (\delta J_o^{sh} + (1-\delta) J_o^{sl})) \\
& \quad + \frac{1}{2} (\beta (\delta J_o^{uh} + (1-\delta) J_o^{ul}))
\end{aligned}$$

Employed old worker

An employed old worker is the one for whom the match from the last period is intact. In the Baseline case, the employed worker prefers to work instead of enjoying leisure. Therefore, the surplus of an employed worker is merely the wages net of the utility from leisure that he forgoes in order to work and is given by $(w_{o,t}^{eh} - l_h)$ and $w_{o,t}^{el}$. For the firm, depending on whether the worker is of high or low leisure type, the surplus is given by $(p - w_{o,t}^{eh} + \beta \pi_{v,t+1} - \beta \pi_{v,t+1})$ and $(p - w_{o,t}^{el} + \beta \pi_{v,t+1} - \beta \pi_{v,t+1})$. Bargaining with equal weights implies that the aggregate surplus is equally shared between the agents. Hence the wages for the employed worker with high and low preference for leisure in the *steady state* are $w_o^{eh} = \frac{p+l_h}{2}$ and $w_o^{el} = \frac{p}{2}$ respectively. Firms matched with workers from the previous period do not incur any search and hiring costs. This affects the total surplus from the match and in turn affects the wages earned by the old employed workers.

Separated old worker

A separated worker is the one who was matched with a firm when young but the match dissolved at the end of the first period. His preference for leisure is revealed before he re-enters the market in search for a job. His surplus from employment in the Baseline Case, based on his leisure preference is either $(w_{o,t}^{sh} - l_h)$ or $w_{o,t}^{sl}$. The firm when matched with a separated worker has to incur hiring costs in addition to the search cost. The cost of hiring a separated worker is lower compared to new hires, young and never-before employed workers. The surplus, based on different leisure preferences of the worker, is the value output net of the hiring and wage costs and is given by $(p - h_s - w_{o,t}^{sh} + \beta\pi_{v,t+1} - \beta\pi_{v,t+1})$ or $(p - h_s - w_{o,t}^{sl} + \beta\pi_{v,t+1} - \beta\pi_{v,t+1})$. Nash Bargaining with equal weights implies that the wages of the two types of separated workers in the *steady state* are $w_o^{sh} = \frac{p-h_s+l_h}{2}$ and $w_o^{sl} = \frac{p-h_s}{2}$. Here due to the job destruction after one period of employment, both agents incur search costs in addition to the hiring costs incurred by the firms. This translates into lower surplus for both workers compared to the employed worker case.

Unemployed old worker

The never-before employed worker entered the labor market when young but was not matched with a firm. He re-enters the labor market in the following period. The worker's preference for leisure is revealed before he enters the market in the second period of his life. The surplus enjoyed by the two types of workers if they get matched with a firm are given by $(w_{o,t}^{uh} - l_h)$ and $w_{o,t}^{ul}$. For the firm, a match with either a high or low leisure type of a never-before employed worker results in the following surplus: $(p - h - w_{o,t}^{uh} + \beta\pi_{v,t+1} - \beta\pi_{v,t+1})$ or $(p - h - w_{o,t}^{ul} + \beta\pi_{v,t+1} - \beta\pi_{v,t+1})$. The wages of the two workers in the *steady state* are $w_o^{uh} = \frac{p-h+l_h}{2}$ and $w_o^{ul} = \frac{p-h}{2}$.

Therefore, a comparison of the wages of all the old workers indicates the importance of employment histories and leisure preference in the determination of the total surplus accruing from a match. The wages for an old worker, employed, separated or never-before employed, are independent of the endogenous variables, α and θ , and depend on the market value of the firms output, the hiring costs and the utility from leisure. Differences in the hiring costs are due to different employment histories. For example, a firm does not have to incur any hiring costs when it is matched with an old employed worker from the last period. A match with a separated worker reduces the surplus of a firm as they have to incur hiring costs and this reduces the wage of a separated worker relative to an employed worker. A match with a never-before employed worker further reduces the surplus for the firm as the cost of hiring a separated worker is less than hiring a never-before employed worker. Different leisure preferences among the old workers in turn affects the bargaining power of workers. An old worker with high preference for leisure has to be compensated in order for them to give up leisure to work. Therefore, leisure preferences further differentiate the workers that have the same employment history.

2.7 Stationary Equilibria

This section describes the steady state equilibrium for the Baseline case. The aggregate equilibrium state is one where firms and workers maximize their respective objective functions subject to the matching technology. In the steady state, there is no population growth and the young and old workers are of equal measure. Therefore, the flow of workers into unemployment equals the flow of workers out of unemployment.

Definition 1 *A steady -state equilibrium of the Baseline case with free entry in the labor market consists of wage functions w_y^* , w_o^{eh*} , w_o^{el*} , w_o^{sh*} , w_o^{sl*} , w_o^{uh*} , w_o^{ul*} , exogenously specified*

bargaining weights ($\frac{1}{2}$) for the worker and the firm, and a quadruple $(\alpha^*, \theta^*, U^*, F_v^*)$ satisfying the following conditions: (i) Nash bargaining; (ii) Unrestricted entry for firms: $\pi_v^* = 0$; and (iii) Steady-State: $\alpha^*.U^* = \theta^*.F_v^* = \mu M(U^*, F_v^*)$. The equilibrium in this case requires additional restrictions that ensure that all the agents participate in the labor market i.e., $J_y^* > \beta(\delta l_h + [1 - \delta] l_l)$, $J_o^{eh*} > l_h$, $J_o^{el*} > l_l$, $J_o^{sh*} > l_h$, $J_o^{sl*} > l_l$, $J_o^{uh*} > l_h$, and $J_o^{ul*} > l_l$.

The Steady-State Matching Condition

The matching condition for the economy is given by:

$$\alpha.U = \theta F_v = m = \mu(U)^\phi (F_v)^{1-\phi}$$

where $\phi \in [0, 1)$. Noting that $\frac{F_v}{U} = \frac{\alpha}{\theta}$, the steady state matching condition can be written as:

$$\theta^* = \left(\frac{\mu}{\alpha^{*\phi}} \right)^{\frac{1}{1-\phi}} \quad (2.6)$$

Note that the steady-state matching condition implicitly defines a relationship between α and θ consistent with the steady-state values for U and F_v . The dependence of α and θ on the relative number of unemployed workers and firms with vacancies indicates the presence of trading externalities. An additional hiring firm makes workers seeking employment better off as it increases the probability of getting matched with a firm, but it makes other hiring firms worse off as it decreases the probability of a firm getting matched with a worker. The same is true for workers. An additional worker seeking employment makes hiring firms better off but other searching workers worse off. Thus the SS locus is downward sloping (Figure 2).

The Equilibrium Entry Condition

Firms choose to enter the labor market in order to search for an employee and continue to do so until all profit opportunities from new jobs are driven to zero. Thus the equilibrium

entry condition implies

$$\begin{aligned}
a &= \theta \tilde{u}_y [(p - h - w_y)] \\
&+ \beta (1 - b) \{ \delta (p - w_o^{eh}) + (1 - \delta) (p - w_o^{el}) \} \\
&+ \theta \tilde{u}_s [\delta (p - h_s - w_o^{sh}) + (1 - \delta) (p - h_s - w_o^{sl})] \\
&+ \theta \tilde{u}_o [\delta (p - h - w_o^{uh}) + (1 - \delta) (p - h - w_o^{ul})]
\end{aligned} \tag{2.7}$$

Recall that the wages for all the old workers are $w_o^{eh} = \frac{p+l_h}{2}$, $w_o^{el} = \frac{p}{2}$, $w_o^{sh} = \frac{p-h_s+l_h}{2}$, $w_o^{sl} = \frac{p-h_s}{2}$, $w_o^{uh} = \frac{p-h+l_h}{2}$, $w_o^{ul} = \frac{p-h}{2}$. Substituting these wages in equation (2.7) gives the following equilibrium entry condition

$$\begin{aligned}
a &= \theta \tilde{u}_y [(p - h - w_y) + \beta (1 - b) \frac{p}{2}] + \theta \tilde{u}_s [\frac{p - h_s}{2}] \\
&+ \theta \tilde{u}_o [\frac{p - h}{2}] - \frac{\delta l_h}{2} [\theta \tilde{u}_y \beta (1 - b) + \theta \tilde{u}_s + \theta \tilde{u}_o]
\end{aligned} \tag{2.8}$$

where the wage for a young workers under the equilibrium condition that $\pi_v = 0$ is

$$\begin{aligned}
w_y &= \frac{(p - h)}{2} + \frac{1}{2} (\beta (1 - b) (\delta \pi_f^h + (1 - \delta) \pi_f^l)) - \frac{1}{2} (\beta (1 - b) (\delta J_o^{eh} + (1 - \delta) J_o^{el})) \\
&- \frac{1}{2} (\beta b (\delta J_o^{sh} + (1 - \delta) J_o^{sl})) + \frac{1}{2} (\beta (\delta J_o^{uh} + (1 - \delta) J_o^{ul}))
\end{aligned} \tag{2.9}$$

Nash Bargaining with equal weights implies that the surplus is equally shared between the two agents. The surplus in the first period from hiring a young worker is $(p - h)$ and it is equally shared between the two agents. If the match stays intact after the first period, the young worker gets his share of the expected discounted profits $(\beta (1 - b) (\delta \pi_f^h + (1 - \delta) \pi_f^l))$ of the firm accruing in the second period. If a young worker gets matched with a firm, then in the following period he can be in the following two states: employed or separated. Therefore, the firm extracts from a young worker it's share of the gains in expected utility accruing to the young worker in the following period if the match stays intact

$(\frac{1}{2} (\beta (1 - b) (\delta J_o^{eh} + (1 - \delta) J_o^{el})))$ or dissolves $(\frac{1}{2} (\beta b (\delta J_o^{sh} + (1 - \delta) J_o^{sl})))$. If instead the young worker does not find employment and re-enters the labor market in the following period, his share of the expected discounted utility would be $\frac{1}{2} \beta (\delta J_o^{uh} + (1 - \delta) J_o^{ul})$.

Rewriting equation (2.9) after substituting for the expected profits and expected utilities gives the following equation for the wages of a young worker

$$w_y = \frac{(p - h)}{2} - \frac{\beta s (1 - b)}{2} - \beta (1 - b) \frac{\delta l_h}{2} \left[\frac{\alpha}{2} \right] \quad (2.10)$$

$$- \frac{1}{2} \left(\beta b \alpha \left(\frac{p - h_s}{2} \right) \right) + \frac{1}{2} \left(\beta \alpha \left(\frac{p - h}{2} \right) \right)$$

Ceteris paribus, a higher search cost (s higher) is associated with lower wages to the young because it lowers their option value (threat point) of waiting to find employment in the future. In other words, it makes the current job offer more attractive. The firm therefore takes its share of the surplus it creates for the worker by offering them this job. Increase in the hiring costs of new hires (h increases) lowers the wages of the young worker because it decreases the expected profits of the firm which in turn lowers the total surplus from a match with new hires. A higher cost of hiring a separated worker (h_s increases) is associated with higher wages. This is because the firm compensates the young worker for the future loss of income. An increase in the utility from leisure (l_h increases), implies that the expected profits of the firms will decline as the high types of workers would require greater compensation to participate in the labor market. This adversely affects the wages of the young worker. Finally, an increase in probability of being high leisure type (δ increases) decreases the expected profits of the firms as a larger fraction of the population would require higher compensation to participate in the market. This lowers the total surplus from a match with a young worker and therefore lowers his wage.

Substituting equation (2.10) and the expressions for the fraction of each type of unemployed in equation (2.7), the closed form expression for the equilibrium entry condition

is

$$\alpha^* = \frac{2(p-h) + \beta(1-b)(s+p-\delta l_h) - \delta l_h - \frac{4a}{\theta^2}}{\left(1 + \frac{\beta}{2}\right) \{(p-h-\delta l_h) - b(p-h_s-\delta l_h)\} - \frac{2a}{\theta^2}(1-b)} \quad (2.11)$$

Ceteris paribus, an increase in the probability with which a firm finds an unemployed worker (θ higher) increases the expected profits of posting a vacancy. This in turn encourages firm entry which increases the probability of a worker finding employment (α increases). Therefore, the EE locus is upward sloping (Figure 2).

2.8 Numerical Experiments in the Baseline Model

This section illustrates a benchmark example for the Baseline Case given a set of parameters. The results of this benchmark example are compared to that of B&R, highlighting the differences between the two and the impact of these differences on the labor market.

Example 1 *The parameters of the economy are as follows: $\beta = 0.9$, $s = 0.05$, $a = 0.2$, $\mu = 0.4$, $\phi = 0.5$, $b = 0.3$, $p = 1$, $h = 0.3$, $h_s = 0.21$, $\delta = 0.2$ and $l_h = 0.3$. Then, it can be easily checked that equations (2.6) and (2.11) have a unique and economically meaningful solution given by $(\alpha^*, \theta^*) = (0.400, 0.400)$. At this equilibrium, $w_y^* = 0.372$, $w_o^{eh*} = 0.65$, $w_o^{el*} = 0.5$, $w_o^{sh*} = 0.545$, $w_o^{sl*} = 0.395$, $w_o^{uh*} = 0.5$, $w_o^{ul*} = 0.35$, $J_y^* = 0.324$, $J_o^{eh*} = 0.65$, $J_o^{el*} = 0.5$, $J_o^{sh*} = 0.348$, $J_o^{sl*} = 0.108$, $J_o^{uh*} = 0.330$, $J_o^{ul*} = 0.090$, $\tilde{u}_y^* = 0.581$, $\tilde{u}_s^* = 0.070$, $\tilde{u}_o^* = 0.349$, $F_v^* = 0.860$, $F_f^* = 0.140$, $F^* = 0.999$*

Prior to the comparison of the results of the benchmark example to that of B&R, I highlight here the aspects of this model that differ from that of B&R. The set of parameters used here are identical to theirs. However, in this model, the separated workers are modelled explicitly unlike B&R where the unemployed and separated are combined together. Here the distinction is made through hiring costs. The firms have to incur lower hiring costs ($h > h_s$) when they are matched with a separated worker. The old workers apart from having different

employment histories, also differ based on their preference for leisure. A worker with a particular employment history is further classified as being high or low leisure type. Therefore, this implies that the results of B&R can be retrieved from this model by setting $h = h_s$, $\delta = 0$ and $l_h = 0$.

Table 1 gives the results of the benchmark example. Comparing the results of the benchmark example of the Baseline case to that of B&R, it can be seen that here the probability with which a worker gets matched with a firm decreases (α^* lower) and the probability with which a firm gets matched with a worker increases (θ^* higher). This is because in this model, the bargaining power of the old workers increases. The old workers with high preference for leisure have an option outside the labor market. This requires the firms to pay these workers higher wages in order to compensate them for the loss in leisure due to working. Therefore in equilibrium there are fewer firms as compared to B&R. This in turn decreases the probability with which a worker gets matched with a firm. The total number of unemployed workers is also higher compared to B&R. Consequently, the labor market is tighter vis-a-vis B&R. Lower hiring costs for the separated workers translates into higher wages for the separated workers. The wages of all the workers except the young, is atleast as high as in the B&R model. The wages of a young worker are adversely affected by the increase in the bargaining power of the worker in the following period, i.e., when old.

Given the set of parameters of the benchmark example, it would be insightful to see the effect on the aggregate labor market of varying each parameter in isolation. The comparative results are in Table 2.

For given wages, a higher probability of being high leisure type (δ increases) means that a larger fraction of the old have high preference for leisure. This directly affects the profitability of firms and therefore reduces the number of firms that enter the market to post

vacancies. For a worker, the probability of getting matched decreases (α^* lower). For the firms, the probability of getting matched with a worker increases (θ^* higher). The wages of the young worker decrease. This is because, with greater probability the young worker will have a higher bargaining power when old.

An increase in the utility from leisure (higher l_h) has the same impact on the probabilities of getting matched. Here, the wages of all the high leisure type workers increase due to the increased compensation for the loss in leisure.

An increase in the hiring costs of the new hires (h higher) decreases the expected profits of the firms. Fewer firms enter the market resulting in the same impact on the probabilities as the earlier two cases. The wages of the new hires, young workers and never-before employed old workers decrease. The firm earns lower profits when matched with these workers and this attributes to the decline in wages of these workers.

An increase in the hiring costs of the separated workers (higher h_s) results in lower α and higher θ . The wages of the separated workers decrease and that of young workers increase. The firm compensates the young worker for this future loss of income.

2.9 Measure of Welfare

The workers here differ in their employment histories and preference for leisure. The expected lifetime utility of a representative young worker J_y , is the summation of the expected utility of the worker in each possible state. Thus, J_y is used for measuring social welfare. In the benchmark example $J_y^* = 0.324$. The welfare is higher here compared to B&R where $J_y = 0.298$. Welfare, as measured by J_y^* is higher compared to B&R, even though the probability with which a worker gets matched with a firm (α^*) and the wage earned by a young worker (w_y^*) are lowering this case compared to B&R. This is due to two reasons.

Firstly, the separated workers earn higher wages due to lower hiring costs. Secondly the wages of the never-before employed and employed old workers is at least as high as in B&R. This effect dominates resulting in increased welfare.

Table 3 gives the results of the change in welfare when each parameter is varied in isolation and compares the welfare change with B&R.

Comparing the welfare change, it can be seen that the direction of change in welfare is that same in the two models. An increase in either the probability that a match dissolves after one period, or the cost of search for a firm or the cost of search for a worker, decreases welfare. This is because for the first two cases the profits of a firm decrease and the latter is due to a decrease in the expected utility of the worker.

Table 4 highlights the change in welfare as result of a change in the utility derived from leisure (l_h) or the probability of being high leisure type (δ).

Welfare increases in both cases, an increase in the utility from leisure or an increase in the probability of being the high leisure type. The probability that a worker gets matched with a firm decreases in either case. This is offset by an increase in utility from leisure or an increase in the probability of getting high utility from leisure.

Therefore, the Baseline case, where the pattern of labor market participation was such that all the seven types of worker participated in the labor market, indicates the following effects on the labor market. Firstly, an increase in the bargaining power of old workers through their preference for leisure adversely effects the labor market by lowering firm's profits and in turn decreasing the number of firms that participate in the labor market. Secondly, displaced workers earn lower wages than the employed old workers but higher than the never-before employed workers, indicating that employment history has an effect on the wages that the

workers draw in equilibrium. Thirdly, old workers with high preference for leisure are compensated for their loss in leisure in order for them to supply labor.

Chapter 3

True Retirement Case

In the Baseline case, discussed in the previous chapter, all the seven types of workers participated in the labor market. The following model differed in two ways from the B&R model. Here the cost of hiring a separated worker is lower than the hiring cost of the old never-before employed workers. Intuitively, the work experience acquired by the separated workers when young enables them to be re-employed the following period with less amount of training. In addition, old workers have a preference over leisure. Therefore, the Baseline case apart from addressing the inter-generational conflicts of the labor market, deals with the aggregate effects of leisure on the labor market. The results of the benchmark example of the baseline case, relative to B&R indicate that in this case the labor market is tighter and therefore the probability with which a worker gets matched with a firm is lower. This is because in the baseline case, the trading externalities existent in B&R have been exacerbated. The additional heterogeneities among the older workers increase the trading externalities. Here all seven types of workers are matched by a single matching technology. The matching of the workers with the firms is followed by Nash Bargaining. The Nash bargaining process is therefore unable to internalize the trading externalities. The preference over leisure increases the bargaining power of older workers and firms have to compensate the workers for the loss in leisure in order for them to supply labor. This erodes the profits of firms and thus fewer firms enter the labor market to search for workers. Therefore, the baseline case suggests that when all the jobs have the same productivity, workers differ in their employment histories and leisure preference and the search intensity is fixed, the critical variable that drives the equilibrium is job creation, which is determined by the profit maximizing firms.

Retirement commonly implies withdrawal from the labor market, complete or partial, of the old agents who are currently employed. Currently employed workers will be motivated to withdraw from the labor market if they have some option outside the labor market. Leisure seems to be an important motivation for leaving the work-force. Recent trends in the labor market indicate that an increasing number of retirees are citing a preference for leisure as their main motivation for leaving the labor force. Costa documents that among the men who retired at age 65, in 1941-1951 only 3% gave leisure to be their reason for retiring. This number rose to 17% in 1963 and 48% in 1982.

Old workers are also induced to withdraw from the labor market through various government policies. Economists have attributed the decline in the labor market participation of older workers to the generosity of the public pension programs.

In this chapter we focus on the old employed workers and ask this hypothetical question: What would be the effect on the labor market if the old employed workers with an outside option withdraw from the labor market? Recall that one of the limitations of the B&R model was that it was unable to capture this group of old employed workers. In this chapter we study the general equilibrium consequences of the voluntary withdrawal of employed old workers with high preference for leisure from the labor market. The withdrawal is not induced and is not an outcome of any public policy.

3.1 Model

In the baseline case, the pattern of labor market participation was such that all the seven types of workers actively participated in the labor market. Here the old employed workers with high preference for leisure withdraw from the work force. Therefore, the workers who were employed when young and their match stays intact in the second period, choose not to

supply labor in the market but instead enjoy leisure if they have high preference for leisure. The gains from staying out of the labor market need not necessarily exceed the gains from actively participating. Also, these workers who voluntarily withdraw from the labor market, do not constitute the mass of unemployed workers.

Given this, the pattern of labor market participation in this case implies that $J_y > \beta(\delta l_h + [1 - \delta] l_l)$, $J_o^{el} > l_l$, $J_o^{sh} > l_h$, $J_o^{sl} > l_l$, $J_o^{uh} > l_h$ and $J_o^{ul} > l_l$. Therefore, a worker's payoff in each state is given by

$$J_y = -s + \alpha[w_y + \beta(1 - b)\{\delta J_o^{eh} + (1 - \delta) J_o^{el}\} + \beta b\{\delta J_o^{sh} + (1 - \delta) J_o^{sl}\}] \\ + (1 - \alpha)\beta\{\delta J_o^{uh} + (1 - \delta) J_o^{ul}\}$$

$$J_o^{eh} = l_h$$

$$J_o^{el} = w_o^{el}$$

$$J_o^{sh} = -s + \alpha w_o^{sh} + (1 - \alpha) l_h$$

$$J_o^{sl} = -s + \alpha w_o^{sl} + (1 - \alpha) l_l$$

$$J_o^{uh} = -s + \alpha w_o^{uh} + (1 - \alpha) l_h$$

$$J_o^{ul} = -s + \alpha w_o^{ul} + (1 - \alpha) l_l$$

Note that here the expected lifetime utility of an old worker with high preference for leisure (J_o^{eh}) is merely the utility he derives from leisure. In the baseline case, this was equal to the wages that the worker drew in the labor market. Therefore, a young worker's expected lifetime

utility from working simplifies to

$$\begin{aligned}
J_y = & -s + \alpha[w_y + \beta(1-b)\{\delta l_h + (1-\delta)w_o^{el}\}] \\
& + \beta b\{\delta(-s + \alpha w_o^{sh} + (1-\alpha)l_h) + (1-\delta)(-s + \alpha w_o^{sl})\} \\
& + (1-\alpha)\beta\{\delta(-s + \alpha w_o^{uh} + (1-\alpha)l_h) + (1-\delta)(-s + \alpha w_o^{ul})\}
\end{aligned} \tag{3.1}$$

Holding α , θ and wages constant, the difference in the expected lifetime utility of a young worker in the Baseline case and the True Retirement case is $\alpha\beta(1-b)\delta(w_o^{eh} - l_h)$. The expected lifetime utility of a young worker is lower in the True Retirement case vis-a-vis the baseline case if $w_o^{eh} > l_h$. This is because an old worker would be better off supplying labor in the market if the wages he drew were higher than the utility he received from leisure. It is assumed that the workers withdraw from the market even when $w_o^{eh} > l_h$.

The total mass of unemployed workers in the True Retirement case is $U_t = u_{y,t} + u_{s,t} + u_{o,t}$ where $u_{y,t} = \frac{1}{2}$. In the *steady state*, $u_s = \frac{\alpha b}{2}$, $u_o = \frac{1-\alpha}{2}$ and $U = \frac{2-\alpha(1-b)}{2}$. Therefore, $\tilde{u}_y = \frac{1}{2-\alpha(1-b)}$, $\tilde{u}_s = \frac{\alpha b}{2-\alpha(1-b)}$ and $\tilde{u}_o = \frac{1-\alpha}{2-\alpha(1-b)}$. The fraction of each type of unemployed worker in the total mass of unemployed in the True Retirement case is identical to the Baseline case. In the model, the old employed workers enter the labor market only when young and at the beginning of the second period, they are already matched with a firm from the previous period. Therefore, the withdrawal of these workers does not affect the mass of unemployed workers.

The firm's problem is identical to the Baseline case except that here $\pi_f^h = \beta\pi_v$. At the end of the first period, $(1-b)$ fraction of the matches stay intact. The old workers' preference for leisure is revealed at the end of the first period and before the beginning of the second period. In the Baseline case, firms matched with a worker at the end of the first period had a filled vacancy in the second period because all the workers participated in the labor market irrespective of their leisure preference. In the True Retirement case, firms matched with old

workers with high preference for leisure have a vacancy in the second period because the workers withdraw from the market. In the second period, these firms have to search for new hires and thus incur search and hiring costs. The expected discounted profits in the True Retirement case are

$$\begin{aligned} \pi_v = & \frac{1}{1 - \theta \tilde{u}_y \beta (b + (1-b)\{\delta + (1-\delta)\beta\}) - \theta \tilde{u}_s \beta - \theta \tilde{u}_o \beta - (1-\theta)\beta} \{-a \quad (3.2) \\ & + \theta \tilde{u}_y [(p - h - w_y) + \beta (1-b) \{(1-\delta)(p - w_o^{el})\}] \\ & + \theta \tilde{u}_s [\delta (p - h_s - w_o^{sh}) + (1-\delta)(p - h_s - w_o^{sl})] \\ & + \theta \tilde{u}_o [\delta (p - h - w_o^{uh}) + (1-\delta)(p - h - w_o^{ul})] \} \end{aligned}$$

Holding α , θ and wages constant, the expected profits in the True Retirement case are lower compared to the baseline case. This is because, δ fraction of the firms matched with workers from the previous period now have an unfilled vacancy. Therefore, these firms have to incur search costs and if they get matched with a new hire, they have to incur hiring costs. Therefore the expected profits of a firm decline compared to the baseline case.

The flow of vacancies for a firm into period $t+1$ is given by $F_{v,t+1} = F_{f,t} + (1-\theta)F_{v,t} + \theta \tilde{u}_{y,t} b F_{v,t} + \theta \tilde{u}_{y,t} (1-b) \delta F_{v,t} + \theta (\tilde{u}_{s,t} + \tilde{u}_{o,t}) F_{v,t}$ where $F_{f,t+1} = \theta \tilde{u}_{y,t} (1-b) (1-\delta) F_{v,t}$. Accounting restrictions require that $F_{f,t} + F_{v,t} = F_t$. In the *steady state* the masses of firms are: $F = \frac{\alpha(1-b)(1-\delta)}{2} + \frac{\alpha}{2\theta} [2 - \alpha(1-b)]$; $F_v = \frac{\alpha}{2\theta} [2 - \alpha(1-b)]$; $F_f = \frac{\alpha(1-b)(1-\delta)}{2}$. The number of firms with filled vacancies in the True Retirement case is $(1-\delta)$ times the Baseline case. This is because in the True Retirement case, δ fraction of the old employed workers, workers with high preference for leisure, withdraw from the market. The firms with which they were matched no longer have a filled vacancy. Therefore in this case, for given α and θ there will be fewer firms with filled vacancies and hence fewer total number of firms in the labor market.

The matching technology is the same as before. Recall that the matching technology is a function of the aggregate number of workers looking for a job and the number of firms looking for workers. In the True Retirement case, the expected profits of firms are lower compared to the Baseline case. This implies that in equilibrium fewer firms will post vacancies. Therefore, one would expect the probability with which a worker gets matched with a firm to be lower in the True Retirement case as fewer firms participate in the labor market.

Nash bargaining process with equal weights determines the wage functions for all the workers. The wages of all the old workers are identical to the baseline case and are given by $w_o^{el} = \frac{p}{2}$, $w_o^{sh} = \frac{p-h_s+l_h}{2}$, $w_o^{sl} = \frac{p-h_s}{2}$, $w_o^{uh} = \frac{p-h+l_h}{2}$ and $w_o^{ul} = \frac{p-h}{2}$. This is because in the True Retirement case, the withdrawal of the old employed workers with high leisure preference leaves the total match surplus of the remaining old workers and the firms matched with these workers unaltered. The wage functions for the old workers are independent of the endogenous variables (α and θ) and are a function of the market value of the firms output, the hiring costs and the utility from leisure which are exogenously given.

In the True Retirement case, the surplus accruing to both agents from a match with a young worker is different from the baseline case. The surplus accruing to a young worker from seeking employment differs in the second period. Instead of working when the match stays intact, the worker withdraws from the labor market in the second period if he happens to be the high leisure type. The expected lifetime utility of an old employed with high preference for leisure is merely the utility that he derives from leisure, i.e., $J_o^{eh} = l_h$. In the baseline case, $J_o^{eh} = w_o^{eh}$ because the worker chose to work instead of enjoying leisure. For the firm, if the match stays intact with probability $(1 - b)$, it will have a vacancy the following period if the old employed worker has high preference for leisure. Nash bargaining with equal weights

implies that the total match surplus is equally shared between the two agents. Referring back to equation (2.5), in the True Retirement case Nash bargaining implies

$$\begin{aligned} & w_y + \beta(1-b)\{\delta l_h + (1-\delta)w_o^{el}\} + \beta b\{\delta(-s + \alpha w_o^{sh} + (1-\alpha)l_h) \\ & + (1-\delta)(-s + \alpha w_o^{sl})\} - \beta\{\delta(-s + \alpha w_o^{uh} + (1-\alpha)l_h) + (1-\delta)(-s + \alpha w_o^{ul})\} \\ & = (p - h - w_y) + \beta b\pi_v + \beta(1-b)\{\delta\pi_v + (1-\delta)(p - w_o^{el} + \beta\pi_v)\} - \beta\pi_v \end{aligned}$$

The wage function for a young worker is therefore

$$\begin{aligned} w_y &= \frac{(p-h)}{2} - \frac{1}{2}(\beta\pi_v(1-b)(1 - \{\delta + (1-\delta)\beta\})) - \frac{\beta s(1-b)}{2} \quad (3.3) \\ & - \beta(1-b)\frac{\delta l_h}{2}\left(\frac{\alpha}{2}\right) - \frac{1}{2}\left(\beta b\alpha\left(\frac{p-h_s}{2}\right)\right) + \frac{1}{2}\left(\beta\alpha\left(\frac{p-h}{2}\right)\right) \end{aligned}$$

Holding α constant, the difference between the wage of a young worker in the baseline case and the True Retirement case is given by $(1-\beta)(\pi_v^{TR} - \pi_v^B) - \pi_v^{TR}\delta(1-\beta)$. Recall that $(\pi_v^{TR} - \pi_v^B) < 0$ and hence $(w_y^B - w_y^{TR}) < 0$. Therefore, a young worker's wage is lower in the True Retirement case compared to the Baseline case. A decrease in the job durability due to withdrawal of old employed workers with high preference for leisure adversely affects the profits of the firms which in turn lowers the wages earned by young workers.

3.2 Stationary equilibria

Definition 2 *A steady-state equilibrium with free entry in the labor market consists of wage functions $w_y^{\cdot}, w_o^{el^{\cdot}}, w_o^{sh^{\cdot}}, w_o^{sl^{\cdot}}, w_o^{uh^{\cdot}}, w_o^{ul^{\cdot}}$, exogenously specified bargaining weights ($\frac{1}{2}$) for the worker and the firm, and a quadruple $(\alpha^{\cdot}, \theta^{\cdot}, U^{\cdot}, F_v^{\cdot})$ satisfying the following conditions: (i) Nash bargaining; (ii) Unrestricted entry for firms: $\pi_v^{\cdot} = 0$; and (iii) Steady-State: $\alpha^{\cdot} \cdot U^{\cdot} = \theta^{\cdot} \cdot F_v^{\cdot} = \mu M(U^{\cdot}, F_v^{\cdot})$. The equilibrium in this case requires additional restrictions that ensure that all the agents participate in the labor market i.e. $J_y^{\cdot} > \beta(\delta l_h + [1-\delta]l_l)$, $J_o^{el^{\cdot}} > l_l$, $J_o^{sh^{\cdot}} > l_h$, $J_o^{sl^{\cdot}} > l_l$, $J_o^{uh^{\cdot}} > l_h$ and $J_o^{ul^{\cdot}} > l_l$*

The Steady-State Matching Condition is the same as the Baseline case.

$$\theta^* = \left(\frac{\mu}{\alpha^* \phi} \right)^{\frac{1}{1-\phi}} \quad (3.4)$$

Note that the steady-state matching condition is identical to equation (6). Thus the SS locus which is downward sloping is same as the baseline case and does not shift.

In the True Retirement case, firms choose to enter the labor market in order to search for an employee and continue to do so until all profit opportunities from new jobs are driven to zero. Thus the equilibrium entry condition for the True Retirement case simplifies to

$$\begin{aligned} a = & \theta \tilde{u}_y [(p - h - w_y) + \beta(1 - b)(1 - \delta) \frac{p}{2}] \\ & + \theta \tilde{u}_s \left[\frac{p - h_s}{2} - \frac{\delta l_h}{2} \right] + \theta \tilde{u}_o \left[\frac{p - h}{2} - \frac{\delta l_h}{2} \right] \end{aligned}$$

This is analogous to equation (2.8) except for the first term which in the baseline case was $\theta \tilde{u}_y [(p - h - w_y) + \beta(1 - b) \frac{p}{2}]$. Holding α and θ constant, fewer firms enter the labor market in the True Retirement case. In the True Retirement case, profits accruing to each firm are lower compared to the Baseline case. This implies that in equilibrium fewer firms enter the labor market until all the profits are driven to zero. Thus the labor market will be tighter vis-a-vis the Baseline case. The EE locus shifts upwards(Figure 3) relative to the Baseline case.

The equilibrium wages of a young worker are therefore

$$\begin{aligned} w_y = & \frac{(p - h)}{2} - \frac{\beta s(1 - b)}{2} - \beta(1 - b) \frac{\delta l_h}{2} \left(\frac{\alpha}{2} \right) \\ & - \frac{1}{2} \left(\beta b \alpha \left(\frac{p - h_s}{2} \right) \right) + \frac{1}{2} \left(\beta \alpha \left(\frac{p - h}{2} \right) \right) \end{aligned}$$

Recall that the difference in the wages between the two cases was only driven by the difference in profits. In equilibrium when the profits are zero, the wage function of a young worker in the two cases is identical. The wages will differ due to differences in the equilibrium values of α , the probability with which a worker gets matched with a firm.

Substituting the wages of a young worker and the expressions for the fraction of each type of unemployed in the equilibrium entry condition gives the following closed form expression for α , the probability of a worker getting matched with a firm

$$\alpha^* = \frac{2(p-h) + \beta(1-b)(s+p(1-\delta)) - \delta l_h - \frac{4a}{\theta}}{(1 + \frac{\beta}{2}) \{(p-h - \delta l_h) - b(p-h_s - \delta l_h)\} - \frac{2a}{\theta}(1-b)} \quad (3.5)$$

Ceteris paribus, $\alpha^* > \alpha^*$. Therefore, the effect of the withdrawal of the old employed high types on the aggregate labor market is adverse. It lowers the probability with which a worker gets matched with a firm. The job durability is lower in the True Retirement case and hence fewer firms post their vacancies. This is because a lower job durability decreases the profits earned by each firm. Given the matching technology, this in turn decreases α .

3.3 Numerical Experiments in the True Retirement Case

This section illustrates a benchmark example for the True Retirement case given a set of parameters. The results of this benchmark example are compared to that of the Baseline case, highlighting the effect on the aggregate labor market of the withdrawal of old employed workers with high preference for leisure.

Example 2 *The parameters of the economy are as follows: $\beta = 0.9$, $s = 0.05$, $a = 0.2$, $\mu = 0.4$, $\phi = 0.5$, $b = 0.3$, $p = 1$, $h = 0.3$, $h_s = 0.21$, $\delta = 0.2$ and $l_h = 0.3$. Then, it can be easily checked that equations ?? and ?? have a unique and economically meaningful solution given by $(\alpha^*, \theta^*) = (0.379, 0.422)$. At this equilibrium, $w_y^* = 0.370$, $w_o^{el*} = 0.5$, $w_o^{sh*} = 0.545$, $w_o^{sl*} = 0.395$, $w_o^{uh*} = 0.5$, $w_o^{ul*} = 0.35$, $J_y^* = 0.289$, $J_o^{eh*} = 0.3$, $J_o^{el*} = 0.5$, $J_o^{sh*} = 0.343$, $J_o^{sl*} = 0.100$, $J_o^{uh*} = 0.326$, $J_o^{ul*} = 0.083$, $\bar{u}_y^* = 0.576$, $\bar{u}_s^* = 0.066$, $\bar{u}_o^* = 0.358$, $F_v^* = 0.779$, $F_f^* = 0.106$, $F^* = 0.885$*

In the Baseline case, $\alpha(1 - b)$ of the matches are durable for two periods. Whereas in the True Retirement case $\alpha(1 - b)(1 - \delta)$ of the matches are durable. Lower durability of a match in the True Retirement case relative to the baseline case implies that the firms matched with employed old high types of workers have to incur search costs. If they get matched with new hires, they have to incur hiring costs too. These additional costs erode the profitability of each firm and hence fewer firms enter the labor market. The aggregate effect on the labor market is that it lowers the probability with which a worker gets matched with a firm. This difference in the two cases is explicitly captured in the solutions of the benchmark example. Table 5 gives the results of the benchmark example for the True Retirement case.

When jobs have the same productivity and the search intensity is fixed, the critical variable to job creation is determined by the profit maximization of firms. *Ceteris paribus*, firms make lower profits in the True Retirement case. Therefore, fewer firms enter the market searching for workers. This lowers the probability of a young worker getting matched with a firm (α decreases) and increases the probability of a firm getting matched with a worker (θ increases). The wages of all the old workers are independent of the endogenous variables α and θ and therefore are identical in the two cases. Due to a lower probability of getting matched with a firm, a young worker's wage decreases in the True Retirement case relative to the Baseline case. This implies that the durability of a match that has implications on firm's profitability in turn affects the young worker's wage. A decrease in α lowers the expected lifetime utility of all the workers except for old employed workers. The expected utility of the old employed low type workers is independent of α as these workers do not re-enter the labor market in search of jobs. In the True Retirement case, the expected lifetime utility of an old employed high type worker is the utility he derives from leisure.

Given the set of parameters of the benchmark example, like the baseline case, each parameter is varied in isolation to see its effect on the aggregate labor market. The comparative static results for the True Retirement case are in Table 6.

The direction of change as a result of the variation the in parameters is identical in the two cases, but the magnitude of change differs. This is because of the different patterns of labor market participation in the two cases. When the hiring costs (h & h_s) change, both the direction and magnitude of change is more or less same in the two cases. The difference in magnitude is apparent when parameters affecting leisure are altered. In the True Retirement case, the percentage change is twice that of the Baseline case when δ (probability of being high leisure type) changes and half of the Baseline case when l_h (utility from leisure for the high types) changes. This is because an increase (decrease) in δ implies that a larger (smaller) fraction of the employed old workers withdraw from the labor market. This decreases (increases) firm entry which in turn lowers (increases) the probability of a young worker getting matched with a firm. When δ changes, this additional effect in the True Retirement case, increases the magnitude of change. The withdrawal of the old employed high type of workers from the labor market dampens the effect of a change in l_h in the True Retirement case. This is because, for $\alpha(1 - b)\delta$ fraction of the population, the change in the utility from leisure will merely impact an individual's utility without having any aggregate labor market effect.

3.4 Measure of Welfare

The expected lifetime utility of a young worker (J_y) is the measure of social welfare. In the benchmark example in the True Retirement case $J_y^{\wedge} = 0.289$. compared to $J_y^* = 0.324$ in the baseline case. The lower probability of a worker getting matched with a firm reduces

the overall welfare in the True Retirement case. In this case the labor market is tighter due to lower job durability and hence job creation is lower. Table 7 compares the welfare in the two cases when δ and l_h change

A decrease (increase) in the utility from leisure for the high types affects J_y in the True Retirement case in two ways. Firstly, it increases (decreases) the probability of a worker getting matched with a firm. Secondly, it decreases (increases) the wage earned by all the old workers in equilibrium and also lowers the utility of an old employed high type of worker. The second effect dominates resulting in overall decrease in welfare. The percentage change in welfare is higher in the True Retirement case relative to the baseline case due to direct impact of l_h on the expected lifetime utility of an old employed high type of worker. In the Baseline case, it affects the expected lifetime utility of an old employed high type of worker through changes in his wages. A decrease in δ on the other hand increases welfare in the True Retirement case but decreases welfare in the Baseline case. In the True Retirement case, a decrease in δ apart from decreasing the wage cost of firms matched with old high types of workers has an additional effect. It also implies that more jobs will be durable as fewer workers will withdraw from the market. Therefore, the percentage increase in α due to a decrease in δ is almost twice that of the Baseline case. In addition, a decrease in δ implies that with a lower probability an old worker will be of high type and thus with lower probability he will draw higher wages in the labor market. In the Baseline case this effect dominates lowering the overall welfare. In the True Retirement case, the increase α more than compensates for the latter effect therefore resulting in an increase in welfare when δ decreases.

Chapter 4

Conclusion

This chapter summarizes the results of the earlier chapters and discusses the extensions that would be useful in further understanding the interactions in the labor market.

The trend in the labor market suggests that there are six important conclusions that can be drawn. Firstly, the labor force is getting older. Secondly, an individual's age plays a significant role in determining his labor market status: employed, displaced or separated and unemployed. Thirdly, labor force participation rates of older workers have been steadily declining during the post-war period. Fourthly, increasing number of workers are citing a preference for leisure as their main motivation for leaving the labor force. Fifthly, displacement of the older workers has risen disproportionately in the recent years. Sixthly, many public policies affect the labor market participation decision of older workers – they encourage retirement.

Given these trends, this study incorporates the life-cycle of workers in a search and matching framework. It differentiates between workers mainly through their life-cycle. This results in differences in employment histories. In addition, older workers have preference over leisure which gives them an option outside the labor market.

A study of the Baseline case shows the impact of leisure on the aggregate labor market. Leisure affects the bargaining power of older workers. The high leisure type of workers have a higher bargaining power implying that firms have to compensate them for their loss in leisure. The increased bargaining power of workers in turn affects firm entry as it lowers the profits earned by each firm. Therefore, job creation in the labor market is affected by the bargaining power of workers. The True Retirement case, on the other hand highlights the impact of job durability on the labor market. With the withdrawal of old employed high type of workers,

jobs are less durable compared to the baseline case. Lower job durability therefore affects the profits of the firms and results in lower job creation. The main factors affecting the outcomes of the labor market are the life-cycle of agents and their bargaining power. These factors affect the labor market through the profit maximizing behavior of the firms. Thus the number of firms that actually participate in the labor market to search for workers depends on the profits accruing to each individual firm. The matching function in turn determines the probability with which a worker gets matched with a firm and the probability with which a firm gets matched with a worker.

In conclusion, the current framework is easily amenable to a study of policy induced retirement. This would give useful insights into the impact of these policies on the young and old workers and the aggregate labor market. There are a few other possibilities of extending the model. First, the model could be extended to consider the implications of *endogenous* search effort at each stage of the life-cycle. Such a model may be able to rationalize the incidence of long-term unemployment among the old – young workers (due to the possibility of retaining employment for multiple periods) will have the strongest incentives to search for jobs while old workers will devote less effort to job search. Public pension programs will intensify these incentives, further exacerbating the incidence of long-term unemployment for older workers. Second, richer insights into the labor supply decisions of workers at each stage of the life-cycle can be obtained by incorporating both a labor market participation decision and the choice of hours of work. As in Pissarides (2001), workers and firms will engage in Nash bargaining over *both* wages and hours of work. Incorporating these additional aspects of labor supply will provide additional insights into hours of work over the life-cycle, the evolution towards retirement, the transition to part-time work among older workers, and the aggregate labor market implications of various work-sharing programs. Another possibility

will be to further consider the labor market dynamics in our framework. It would be interesting to consider how the dynamics of the overlapping generations framework compare to standard infinite-horizon models such as in Mortensen (1999). It is plausible that endogenous labor market participation choices at each stage of the life-cycle will affect aggregate labor market outcomes through their implications for dynamics. Finally, further work could consider the simultaneous interactions between health investment choices by workers and labor market conditions, as well as training and human capital investment choices at each stage of the life-cycle.

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Appendix

Table 1: Comparison of Baseline case with B&R

Variable	Baseline case	B & R case
α^*	0.400	0.413
θ^*	0.400	0.387
w_y^*	0.372	0.380
w_o^{eh*}	0.650	N.A
w_o^{el*}	0.50	0.500
w_o^{sh*}	0.545	N.A
w_o^{sl*}	0.395	0.350
w_o^{uh*}	0.500	N.A
w_o^{ul*}	0.350	0.350
J_y^*	0.324	0.298
J_o^{eh*}	0.650	N.A
J_o^{el*}	0.500	0.500
J_o^{sh*}	0.348	N.A
J_o^{sl*}	0.108	0.095
J_o^{uh*}	0.330	N.A
J_o^{ul*}	0.090	0.095
F^*	0.999	1.06
F_f^*	0.140	0.145
F_v^*	0.860	0.913

Table 2: Comparative Statics for the Baseline case

Variable	δ	δ	l_h	l_h	h_s	h_s	h	h
	0.1	0.3	0.2	0.4	0.189	0.231	0.25	0.35
α^*	0.409	0.391	0.406	0.394	0.401	0.399	0.417	0.383
θ^*	0.392	0.409	0.394	0.406	0.399	0.401	0.384	0.418
w_y^*	0.375	0.369	0.374	0.370	0.372	0.373	0.403	0.341
w_o^{eh*}	0.650	0.650	0.600	0.700	0.650	0.650	0.650	0.650
w_o^{el*}	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
w_o^{sh*}	0.545	0.545	0.495	0.595	0.556	0.535	0.545	0.545
w_o^{sl*}	0.395	0.395	0.395	0.395	0.405	0.385	0.395	0.395
w_o^{uh*}	0.500	0.500	0.450	0.550	0.500	0.500	0.525	0.475
w_o^{ul*}	0.350	0.350	0.350	0.350	0.350	0.350	0.375	0.325
J_y^*	0.313	0.335	0.316	0.331	0.325	0.323	0.356	0.292
J_o^{eh*}	0.650	0.650	0.650	0.650	0.650	0.650	0.650	0.650
J_o^{el*}	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
J_o^{sh*}	0.350	0.346	0.270	0.427	0.352	0.344	0.352	0.344
J_o^{sl*}	0.111	0.104	0.110	0.106	0.113	0.103	0.115	0.101
J_o^{uh*}	0.332	0.328	0.251	0.409	0.330	0.330	0.344	0.317
J_o^{ul*}	0.093	0.087	0.092	0.088	0.090	0.090	0.106	0.074
F^*	1.037	0.962	1.025	0.975	1.003	0.996	1.072	0.927
F_f^*	0.143	0.137	0.142	0.138	0.140	0.140	0.146	0.134
F_v^*	0.894	0.825	0.883	0.837	0.863	0.856	0.926	0.793

Table 3: Comparison of Welfare with B & R case for different set of parameters

Variable	Baseline case	B & R
$b = 0.25$	0.340	0.317
$b = 0.30$	0.324	0.298
$b = 0.35$	0.307	0.279
$s = 0.00$	0.406	0.380
$s = 0.05$	0.324	0.298
$s = 0.10$	N.A.	0.280
$a = 0.191$	0.340	0.315
$a = 0.20$	0.324	0.298
$a = 0.25$	0.252	0.222

Table 4: Welfare in the Baseline case under different parameters

Variable	Baseline case
$l_h = 0.20$	0.316
$l_h = 0.30$	0.324
$l_h = 0.40$	0.331
$\delta = 0.10$	0.313
$\delta = 0.20$	0.324
$\delta = 0.30$	0.335

Table 5: Comparison of the True Retirement Case with the Baseline Case

Variable	True Retirement case	Baseline case
$\hat{\alpha}$	0.379	0.400
$\hat{\theta}$	0.422	0.400
\hat{w}_y	0.370	0.372
\hat{w}_o^{eh}	N.A.	0.650
\hat{w}_o^{el}	0.50	0.50
\hat{w}_o^{sh}	0.545	0.545
\hat{w}_o^{sl}	0.395	0.395
\hat{w}_o^{uh}	0.500	0.500
\hat{w}_o^{ul}	0.350	0.350
\hat{J}_y	0.289	0.324
\hat{J}_o^{eh}	0.300	0.650
\hat{J}_o^{el}	0.500	0.500
\hat{J}_o^{sh}	0.343	0.348
\hat{J}_o^{sl}	0.100	0.108
\hat{J}_o^{uh}	0.326	0.330
\hat{J}_o^{ul}	0.083	0.090
\hat{F}	0.885	0.999
\hat{F}_f	0.106	0.140
\hat{F}_v	0.779	0.860

Table 6: Comparative Statics for the True Retirement case

Variable	δ	δ	l_h	l_h	h_s	h_s	h	h
	0.1	0.3	0.2	0.4	0.189	0.231	0.25	0.35
$\hat{\alpha}$	0.398	0.360	0.382	0.376	0.380	0.378	0.396	0.362
$\hat{\theta}$	0.402	0.444	0.419	0.425	0.421	0.423	0.404	0.442
\hat{w}_y	0.374	0.367	0.372	0.369	0.370	0.371	0.401	0.339
\hat{w}_o^{eh}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
\hat{w}_o^{el}	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
\hat{w}_o^{sh}	0.545	0.545	0.495	0.595	0.556	0.535	0.545	0.545
\hat{w}_o^{sl}	0.395	0.395	0.395	0.395	0.405	0.385	0.395	0.395
\hat{w}_o^{uh}	0.500	0.500	0.450	0.550	0.500	0.500	0.525	0.475
\hat{w}_o^{ul}	0.350	0.350	0.350	0.350	0.350	0.350	0.375	0.325
\hat{J}_y	0.295	0.284	0.276	0.301	0.290	0.288	0.320	0.259
\hat{J}_o^{eh}	0.300	0.300	0.200	0.400	0.300	0.300	0.300	0.300
\hat{J}_o^{el}	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
\hat{J}_o^{sh}	0.348	0.338	0.263	0.423	0.347	0.339	0.347	0.339
\hat{J}_o^{sl}	0.107	0.092	0.101	0.099	0.104	0.095	0.106	0.093
\hat{J}_o^{uh}	0.330	0.322	0.245	0.406	0.326	0.326	0.339	0.313
\hat{J}_o^{ul}	0.089	0.076	0.084	0.082	0.083	0.082	0.098	0.068
\hat{F}	0.978	0.797	0.897	0.874	0.889	0.882	0.955	0.817
\hat{F}_f	0.125	0.088	0.107	0.105	0.106	0.106	0.111	0.101
\hat{F}_v	0.853	0.708	0.790	0.768	0.782	0.776	0.844	0.715

Table 7: Welfare Comparisons under different parameters

Variable	True Retirement case	Baseline case
$l_h = 0.20$	0.276	0.316
$l_h = 0.30$	0.289	0.324
$l_h = 0.40$	0.301	0.331
$\delta = 0.10$	0.295	0.313
$\delta = 0.20$	0.289	0.324
$\delta = 0.30$	0.284	0.335

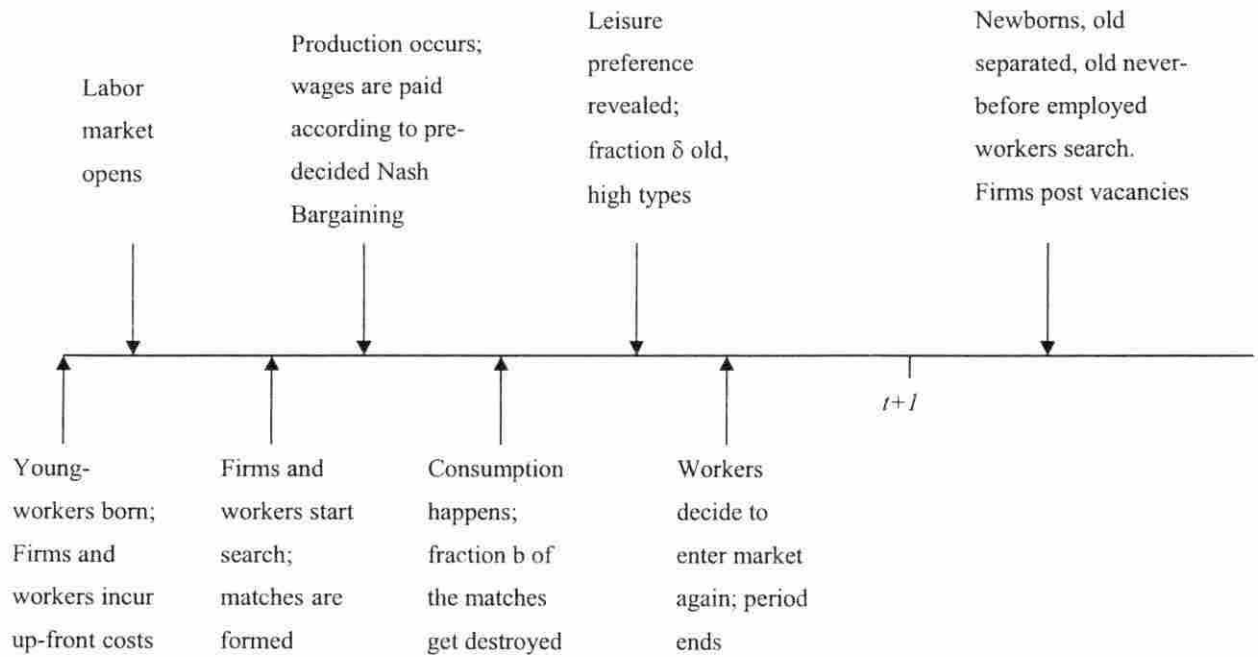


Figure 1: Timeline of events

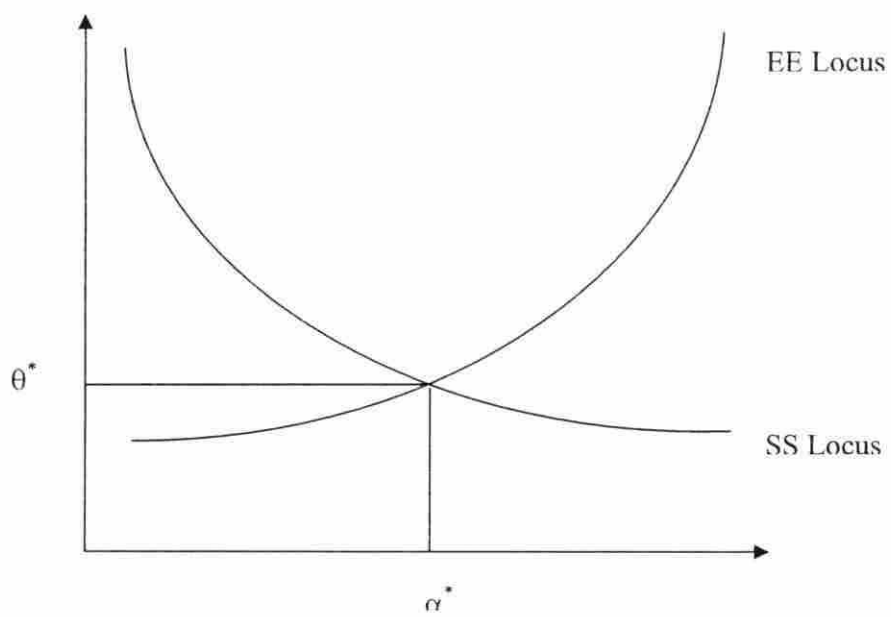


Figure 2: EE and SS Locus for the Baseline Case

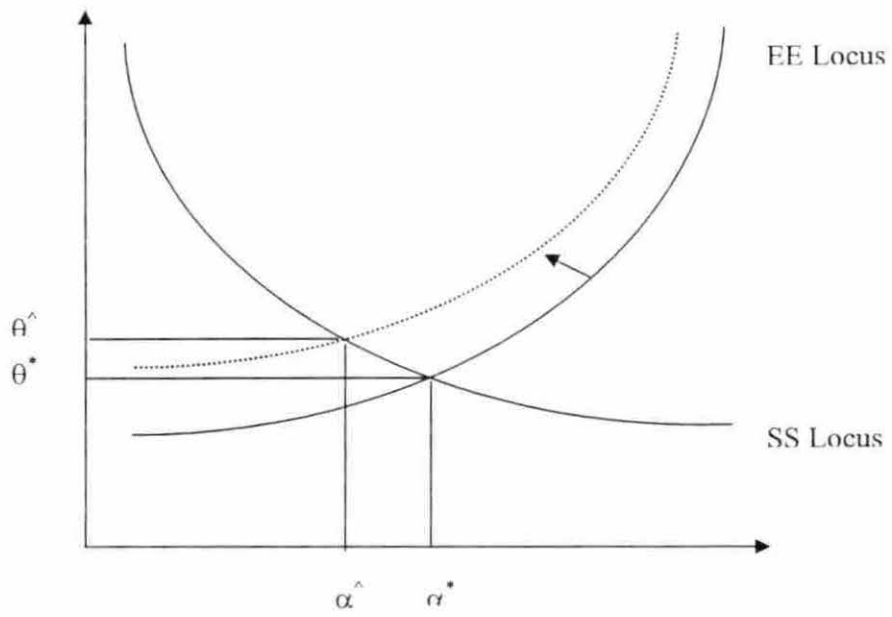


Figure 3: EE and SS Locus for the True Retirement Case